**STAGE 1 MATHEMATICS**

**PROGRAM 4 – SEMESTER 3**

This program is for a cohort of students intending to continue to Mathematical Methods and Specialist Mathematics at Stage 2. The following program describes the third semester of learning.

**SEMESTER THREE - 17 WEEKS INCLUDING EXAM WEEK**

It is assumed this component will be taught after students have completed Program 1 – Semester 1 covering Topics 1, 2 and 3.

* Topic 9 – Vectors in the Plane Subtopics 9.1, 9.2, 9.3 and 9.4 (7 Weeks)
* Topic 12 – Real and Complex Numbers Subtopics 12.2, 12.3, 12.4, 12.5 (5 Weeks)
* Topic 10 – Further Trigonometry Subtopics 10.1 (part), 10.2 (4 Weeks)

**Topic 9 – Vectors in the Plane (7 weeks) Start Term 2 week 9**

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| **Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 2-9 | 9.1Vector Operations | Representation of vectors in the plane by directed line segmentsVector operations including:* Addition and subtraction
* Scalar multiples
* Applications of scalar multiples: parallel vectors and ratio of division
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| 2-10 | 9.2Component and Unit Vector form | Vectors in the Cartesian Plane* ordered pair notation
* column vector notation
* combination of vectors
* unit vectors
* position vector

Determination of length and direction of a vector from its components | **Investigation****Bezier curves** |
| 3-1 | 9.3Projections | Projection of one vector onto another* The dot (scalar) product
* The angle between two vectors
* Perpendicular vectors
* Parallel vectors
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| 3-2 |
| 3-3 | 9.4Geometric proof using Vectors | Geometric Proofs using Vectors in the planeVarious selections of proofs involving parallelism and perpendicularity that makes use of dot product and scalar multiplesExamples include:* The diagonals of a parallelogram meet at right angles if and only if it is a rhombus
* Midpoints of the sides of a quadrilateral join to form a parallelogram
* The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides
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| 3-4 |
| 3-5 |  | **Revision and SAT 1** | **SAT 1 Part 1**Subtopics 9.1, 9.2, 9.3Calculator permitted**SAT 1 Part 2**Subtopic 9.4No calculator  |

**Topic 12 – Real and Complex Numbers Subtopics 12.2, 12.3, 12.4, 12.5 (5 weeks)**

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| **Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-6 | 12.2Introduction to Mathematical Induction  | Ladder/Dominoes example* Initial statement, inductive step
* Prove results for simple sums, such as 1 + 4 + 9 … + n2 = $\frac{n(n+1)(2n+1)}{6}$ for any positive integer n
* Prove results for arithmetic and geometric series
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| 3-7 | 12.3Complex Numbers | Introduction to the imaginary number $i$, and its definition as $i=\sqrt{-1}$. Exemplify its use in solutions to equations such as $x^{2}+1=0$Introduction to complex numbers: $a+bi$ and defining the real and imaginary componentsOperations with complex numbers, including the use of$ i^{2}=-1$* Addition, subtraction, multiplication and division
* Complex conjugates
* Readdress the quadratic formula in the context of complex number solutions
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| 3-8 | 12.4The Complex Plane (Argand) | Cartesian form on an Argand diagramVector addition in the complex planeComplex conjugates in the complex planeModulus |
| 3-9 | 12.5Roots of Equations | Factorise quadratics into linear factorsUse of quadratic formula involving *i.* |  |
| 3-10 |  | **Revision and SAT 2** | **SAT 2**Subtopics 12.2, 12.3, 12.4, 12.5Calculator permitted |

**Topic 10 – Further Trigonometry Subtopic 10.1 (part), 10.2 (4 weeks)**

*The content of this topic assumes Topic 3 - Trigonometry has been completed*

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| **Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 4-1 | 10.1Further Trigonometric Functions (part) | Solve trigonometric equations of the form y=AsinB(x – C)+D (including cosine and tan versions) finding all solutions. |  |
| 4-2 | 10.2Trigonometric Identities | Consider the sine and cosine functions and their behaviour in the unit circle. Hence develop the following trigonometric relationships:* $\sin(\left(-x\right)=-sin⁡(x))and\cos(\left(-x\right)=cos⁡(x))$ (this is a review from subtopic 3.3)
* $sin^{2}x+cos^{2}x=1$
* $sin⁡(2x)=2\sin(\left(x\right))cos⁡(x)$
* $\cos(\left(2x\right))=cos^{2}x-sin^{2}x=1-2sin^{2}x=2cos^{2}x-1$
* $\sin(\left(\frac{1}{2}x\right))=\pm \sqrt{\frac{1}{2}-\frac{1}{2}cosx}$ and $\cos(\left(\frac{1}{2}x\right))=\pm \sqrt{\frac{1}{2}+\frac{1}{2}cosx}$
* $sin⁡(A\pm B)=sinAcosB\pm cosAsinB$ and

 $\cos(\left(A\pm B\right))=cosAcosB\mp sinAsinB$  |  |
| 4-3 | Derive $Asinx+Bcosx=ksin(x+α)$ and find expressions for $k,cosα and sinα$Consider the reciprocal trigonometric functions: $secθ, cosecθ and cotθ$Sketch graphs and complete simple transformations for each of these |  |
| 4-4 |  | **Revision and SAT 3** | **SAT 3**Subtopic 10.1 (part), 10.2Calculator permitted |
| 4-5/6 |  | **EXAMINATION REVISION / EXAMS** |  |