

## 2021 General Mathematics Subject Assessment Advice

### Overview

Subject assessment advice, based on the 2021 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

### School Assessment

#### Assessment Type 1: Skills and Applications Tasks

Students undertake five skills and applications tasks, including at least one skills and applications task from each of the non-examined topics. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

Questions from past examinations or exemplar skills and application tasks can be used as the basis for questions in skills and applications tasks, however they should be amended so they are not easily recognisable and do not form the majority of questions assessed in any individual task. It is recommended that all questions in the SATs should be set in an appropriate context rather than simple fluency style questions with no context for the problem being given (e.g. solve this set of simultaneous equations).

It is a requirement for moderation that SATs are marked to clearly indicate how much of each mathematical problem a student has been successful in attempting. Marking of all calculations in assessment responses is essential to support the moderation process.

Before uploading materials, teachers should check files for reasonable scan quality and that the work has the correct orientation. A variation form also needs to be submitted if a student did not complete one or more skills and applications tasks.

All nine assessment criteria should be assessed at least once in either the skills and applications tasks or in the mathematical investigation. In particular, RC5, 'forming and testing of predictions' is easier to assess within Assessment Type 2: Mathematical Investigations. If it is assessed in the skills and applications tasks, then students should have plenty of opportunity to meet the specific feature to an A standard.

*The more successful responses commonly:*

- included skills and applications tasks that contained enough complex questions to enable effective differentiation across the grade bands. A complexity guide has been provided to support teachers to identify key questions and key concepts that provide the opportunity for complexity in responses. The document 'Complexity Guide General Mathematics' is available on the website at the following link: <https://www.sace.sa.edu.au/web/general-mathematics/stage-2/support-materials/subject-advice-and-strategies>
- were seen when students had the opportunity for interpretation of the mathematical results in the context of the problem, including discussion of the assumptions and limitations of the results, in all skills and applications tasks

- were seen when skills and applications tasks were marked to clearly indicate how much of each mathematical problem a student had been successful in attempting, identify where errors had been made and where incorrect values lead on to follow-on responses being accurate
- where the tasks provided students with the number of marks a question is worth and the appropriate space to allow them to identify the amount of detail required in the answer
- were seen when students used appropriate notation and rounding in all assessment tasks, in particular when the financial and statistical models SATS were rounded appropriately given the context
- were seen when tasks used appropriate verbs such as state, explain and interpret to guide students to the form of response required as well as strategically placed 'show' questions that allowed students back into a question if they were not able to complete a previous part successfully. An example of a 'show' question is providing the answer, or an approximate answer, to an annuity problem so that students who are not able to find the value can use the provided figure to continue on through following parts of the problem. The instruction to 'show' ensures that students must provide evidence of the method used to find the value and cannot gain marks for simply writing the value they have been given in the question stem.

*The less successful responses commonly:*

- were seen in skills and applications tasks that provided limited opportunities for students to respond to questions of a complex nature. Teachers need to ensure that at least 30% of the marks in each task are composed of questions covering complex concepts or requiring complex processes to solve the questions. Please note that where questions requiring complex processes or concepts are heavily scaffolded to support progress through the solution, the complexity is reduced. Also note that a longer unstructured question may also be made up of both routine and complex processes and hence not deemed all complex
- were evident when the student was not given opportunity for interpretation of the mathematical results in context of the problem across all five skills and applications tasks
- were evident when students were providing generic responses to assumptions and limitations rather than responses in context. An example of this in critical paths is when a student simply states a delay in a task on the critical path rather than the student providing a contextual example of what might cause a delay
- used inappropriate notation and rounding. Specific examples include:
  - Topic 5: Discrete Models: some students did not provide units for the maximum of minimum value found in Hungarian algorithm problems,
  - Topic 3: Statistical Models: evidence of students not rounding to a specified number of decimal places was seen, or students not applying basic rounding rules correctly
- included content that is not a part of the specified subject outline content. Examples of this include: plotting the mean point in statistics; bipartite graphs in discrete models;  $Z$  scores in statistics; and using the incorrect method of calculating the comparison rates by adding all the fees up initially instead of adding them to the payment once calculated in finance. It is important that the teachers reference the subject outline each year to review any changes to the curriculum requirements
- where teachers indicated that they were assessing RC5 - *Forming and testing of predictions* and CT3 - *Application of mathematical models*, yet the students were not given an opportunity to show these skills at an A level, or in some instances opportunities were not provided at all
- where an open topic was included that did not have enough complexity for students to achieve in the higher-grade bands. An example of this is covering measurement, only assessing simple shapes for area, surface area and volume.

## Assessment Type 2: Mathematical Investigations

Students undertake two mathematical investigations with a maximum length of 12 A4 single-sided pages with a minimum of size 10 font. The evidence presented in the two investigations should include key ideas and key concepts from at least two different topics.

Teachers may need to provide support and clear directions for the first investigation. However, the second investigation must be less directed and set within more open-ended contexts.

It is a requirement for moderation that teachers ensure that all mathematical solutions produced by the student in the investigations are marked for accuracy and errors are identified as well as making comments about the written component. This supports both student understanding and the moderation process.

Before uploading teachers should check the file for reasonable scan quality and that the work has the correct orientation. It also assists moderators with the moderation process if both tasks are uploaded in the same file.

*The more successful responses commonly:*

- were in students' responses where the tasks designed had enough scaffolding for students to achieve at the C grade band in the initial parts of the task, but also provided an open-ended section that required students to extend their investigation in a direction of their own choosing. This allowed students to demonstrate their understanding at the higher-grade bands
- included a detailed development and application of a mathematical model beyond the initial model, with enough complexity. A complexity guide has been provided to support teachers to identify key questions and concepts that provide the opportunity for complexity. A link to the document 'Complexity Guide General Mathematics' is provided above
- occurred when students demonstrated a comprehensive understanding of the assumptions made in their investigations, the reasonableness of their results, the limitations of the models they had investigated and the effect that these limitations had on the results
- were seen where students made appropriate predictions before they began calculations and used their calculations to refine future predictions, and discussed the accuracy of their predictions. An example of this process is:
  - students make a prediction of how much money they may save on a home loan if they increase the repayment by \$100
  - calculations are made to find the savings gained through the \$100 increase
  - a comparison is made of the calculated value to their prediction
  - students use their answer to make a more refined prediction about how much they will save if they were to, for instance, double the payment to \$200.
- were presented in an appropriate report format, including relevant headings, tables and graphs to present the key results, as well as tables and graphs labelled and a reasonable size for easy reference
- were in responses where RC2 - *Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations* and RC5 - *Forming and testing of predictions* were easy to find within the student responses. Ways that this can be achieved include using appropriate headings or bolding key words (limitations/assumptions/prediction) or for the teacher to clearly indicate where the student covered these criteria
- where the teacher had clearly marked the mathematical calculations for accuracy, indicating where errors had been made as well as making relevant comments about evidence of the RC features in the written sections of the investigation response. The submission of shaded performance standards indicating the assessment decisions for each of the features assessed in each of the investigations supports the moderation process.

*The less successful responses commonly:*

- were in response to tasks that used the same starting parameters or data, reducing the individuality of responses
- had evidence of all students following the same modelling processes (with the same changes implemented to their model), which indicated excessive teacher scaffolding. This particularly impacted the students in the higher-grade bands as scaffolding reduces complex mathematical modelling to a more routine level
- did not show enough complexity in calculations and model development to reach the A grade level even though the task allowed them to. Examples include:
  - Topic 1: Modelling with Linear Relationships, where students did not cover concepts such as wastage, change of constraints, multiple solutions, or non-integer solutions
  - Topic 3: Statistical Models, where students only looked at residual plots for the exponential model instead of all models or didn't look at the impact of removing an outlier
  - Topic 4: Finance Models where students only changed one variable at a time.

- occurred where the performance standards were applied appropriately to the individual responses, but the overall grade awarded was inconsistent with the combination of levels achieved in the specific features. For example, a student who achieved assessments against the specific features mostly within the B grade band, with a few specific features in the C grade band across the two assessments should achieve an overall (holistic) B– grade, rather than a B grade
- occurred when responses lacked depth in the analysis due to the response providing evidence of a recount of what the student did rather than analysis of the outcomes of the mathematical calculations in the context of the problem. Students should be informed that analysis of the mathematical results should provide:
  - interpretation of the answers in context
  - comparison of results
  - discussion of key findings rather than a recount of what they did and how they did it.
- when the predictions made were irrelevant to the investigation or the predictions made could not actually be tested and compared to actual findings
- when the responses lacked evidence of drawing clear conclusions based on their mathematical findings and didn't provide evidence of their understanding of assumptions and limitations in the models applied to address RC2. For example, in an interest minimisation task students drew the broad conclusion that people should use interest minimisation strategies rather than using their mathematical findings to indicate the best interest minimisation strategies to use based on their mathematical evidence
- responses that had too many repetitive calculations, which meant the student was trying to cram too much into a page. This often led to pages with very dense text with little use of paragraphs, or graphs or other mathematical images made so small that it was impossible to provide relevant information. This often limited the student in developing their model to more complex calculations such as calculations that performed multiple changes at once because they had already reached their maximum page limit. Please advise students that initial calculations for each problem type should be included in the main body of the response. Any additional repetitious calculations made to test different variables can be placed in an appendix with the answer tabled in the main body. Teachers are expected to check the accuracy of values in tables by checking the calculations in the appendices. Moderators will need all values to be marked for accuracy
- where they lacked evidence, in either task, of predictions being made prior to calculations being completed or a further discussion of the accuracy of the predictions
- when an open topic was covered and the task did not allow the students to show enough complexity to achieve in the higher-grade bands.

## External Assessment

### Assessment Type 3: Examination

The evidence in the students' responses to the 2021 exam showed that the vast majority of well-prepared students were able to complete the paper in the time available. It was also evident that some of the issues raised in past years' advice were less of a problem, with fewer students losing marks unnecessarily. For details of these issues teachers are referred to previous years' Subject Assessment Advice documents with specific reference to the General Mathematics examination evaluation provided.

Specifically, this year students overall were found to be better at:

- Giving more succinct answers (i.e. less unnecessary repetition of the question or writing a description of the result of a calculation when not required, etc.).
- Rounding calculated answers more appropriately (i.e. giving answers to sufficient accuracy in the case of interest rates, correlation coefficients, parameters when fitting mathematical models, etc, and rounding to an integer where the context made this appropriate).
- Giving a complete answer to a question when more than one thing was required.

One general area that still gives concern is when students are required to relate their response to the context of the question. Such responses are called for throughout the examination (for instance Qu 4b, 5b(iii), 6d(ii), 9c(ii) and (iii)) and students need to be aware that they must explicitly show this relationship to the context in their answers, not just give 'standard' or generic statements, if they are to gain full marks.

Examination markers aim to award marks for evidence of student understanding in response to examination questions wherever possible, however students should be advised not to cross out their responses or attempted responses to questions in the examination booklet unless they are confident that no part of what is crossed out should be considered by the marker.

If a student crosses out a response and then decides that it was the correct (or most correct) answer, then the student should indicate clearly to the marker which part of their response should be considered. This could be done by circling or highlighting all or part of the response the student wants to be considered and writing "please mark this work". Students do not need to rewrite their answers in this case unless the crossing out has rendered the response unreadable.

## Feedback on specific questions

### Question 1 – Normal distribution

Responses to this question showed that the vast majority of students were comfortable carrying out normal distribution calculations, though not all were aware of the need to express the answer to part b(ii) as an integer.

Most students who identified the correct graph as 'D' in part b(i) were able to follow through with the inverse normal calculation. The most common error was selecting graph 'B' (which had the tail in the wrong direction) with some of these students carrying out a consistent calculation in c(ii). Many of the others did the calculation with the tail in the correct direction showing that they are not confident about the connection between the graphical representation and the calculation for the inverse normal.

A large number of students were able to identify graph 'B' as the correct answer to d(i) with the most common incorrect answer being 'D'. This indicates that many students, while able to show the correct relationship between the means on a graph, still do not understand that a larger standard deviation means that the peak of the graph will be lower.

In part d(ii) the vast majority of students justified the claim using the larger mean of Brand Z, but many fewer were able to explain that while the left tails of the two distributions were about the same the right tail of Z went further up the scale showing that a significant number of Brand Z batteries would last longer than even the best of Brand Y (or something equivalent). Many of these students simply stated that Brand Z had a larger standard deviation which, by itself, is insufficient justification.

### Question 2 – Superannuation

A surprising number of students are still having difficulty with 'simple interest' calculations such as the ones in part (a) in this question and in Question 7(a). The most common error was to find the annual amount rather than the quarterly or monthly payment. In part (b) many students used the annual salary of \$60 430 as the PV, instead of zero, showing a lack of understanding of the situation.

Answers to part (c) showed that while most students grasped the types of influences that would affect the final balance many did not take notice of the fact that it had to increase, stating 'change' rather than specifically an 'increase in interest rate' or 'decrease in fees removed', for example. It is also important to note that some students adopted a 'multiple answer' approach to this question by listing three or more different responses (possibly in the hope that at least two would be correct). They need to be aware that if any of these responses are incorrect they cannot gain full marks for the question, as they have not demonstrated the mathematical understanding needed to distinguish between correct and incorrect answers.

The TVM calculations were generally well done although a significant number of students still had difficulty putting the appropriate positive and negative signs on PV, FV and Pmt values in their working and hence got incorrect answers.

There seemed to be less incidence of students using the incorrect number of weeks in a year in this examination compared to past years.

### Question 3 – Hungarian algorithm

The Hungarian algorithm question in this paper was generally very well done with many students coping quite well with the complexity of part (c). The calculations on the array in part b(ii) were, for the most part, carried out efficiently and accurately. The most commonly lost mark in this section was in part b(iii) with students failing to consider the units of the bids and hence giving a total of \$68 instead of \$6 800.

A pleasing number of students handled the modifications to the original scenario in part (c) with at least partial success. The addition of dummy rows and columns as well as multiple solutions provided a test of their depth of understanding of the interpretation of a final array in this type of problem. Many were able to identify at least one of the two outcomes common to all solutions (Buyer 2 must get item 1 and Buyer 4 gets nothing).

Responses to part (c) (iii) showed that some of these students could also find the possible solutions in the array.

### Question 4 – Linear Regression

The responses to this fairly standard statistics question have identified some areas where students need to take care, although overall the question was reasonably well done.

In part (a) many students gave the correct response of ticking the third statement. Those who ticked the second statement clearly did not know how to interpret either the equation or the graph. Those who ticked the first statement were probably relying on the visual evidence in the graph and did not take the given value of  $r^2$  into account.

Part (b) was particularly poorly done, with less successful students:

- describing the strength and nature of the correlation instead of interpreting the slope
- ignoring the context altogether (e.g. 'as x increases, y decreases')
- not using the values or units from the context (e.g. 'as temperature increases, humidity decreases')
- not interpreting the negative sign (e.g. 'humidity decreases/increases by -4.68 %')

In part (c) some students provided a description (79% of the variation in H... etc.) instead of simply stating the value of  $r^2$ . The key word in this question is 'calculate' – no interpretation was required. Students should identify key words in the question stem to determine what response is required and therefore use their time efficiently.

A significant number of students identified the wrong point as the outlier in part (d). Students should be encouraged to look at the scatterplot of the data already entered in their calculator rather than relying on looking at the numerical data alone to find outliers. The most common error was (12.1, 84), possibly because it was the last entry in the table or else because, although it followed the trend, there was a gap between it and the other points.

Most students, whether they were correct or incorrect in part (d) could follow through with the calculations required in parts (e) and (f). It should be noted that students who did not express their answer to part (e) as an equation did not gain the mark.

The most common error in part (g) was for students to justify their choice by saying it was an extrapolation. As it had already been stated in the question that both choices were extrapolations this response was insufficient. Successful students identified the more extreme extrapolation or stated that negative relative humidity was unreasonable.

### Question 5 – Investment

A great many students, while being able to find the effective rate for Option A, were unable to do so for the flat rate in Option C. The most frequently given incorrect answer was the nominal rate of 2.15% showing that students confused flat rate with annual compounding. There was some evidence of students finding the correct answer using the lengthier 'by hand' calculation method, however most of the successful students used  $N = \frac{1}{4}$  and  $I = 2.15$  in the effective rate menu of their calculator.

There was a small but significant number of students who chose the lowest rate in part a(ii), mistaking the investment for a loan.

The calculations in parts b(i) and (ii) were generally well done although quite a few students forgot to subtract the initial PV of \$5000. A surprising number of students could not answer part b(iii) correctly, thinking the answer had to do with interest rates or something else, rather than the fact that although the payments had doubled, the initial PV had not.

Finding the correct inflation rate in part (c) could either be done directly using a TVM calculation for 'I' or by working backwards from the given choices for the answer. If students used the formula for inflation they could solve for 'i' using the equation solver in their calculator, but there was no requirement to solve the equation by algebraic means. The crucial aspect to this question was rounding in the correct direction to find the maximum rate possible.

### Question 6 – Critical Path Analysis

Student responses to the two questions about critical path analysis in this examination highlighted that the most difficult aspect of this type of problem for students is dealing with dummy links. It is evident that students need more practice with problems involving dummy links so that they become comfortable with their impact when answering CPA questions.

The most common incorrect responses to part (a) either left out task F (ignoring the dummy link) or included A, D, etc. (ignoring the implication of the word 'immediately' in the question).

In the backward scan in part (b) the most common error was to give the value of 17 in the central node instead of the correct value of 14. This is not likely to be because of the dummy link but may have been because of the slight 'backwards' direction of the arrow for task H – students need to take care to notice the direction of the arrows in network diagrams. Making this error did not affect the position of the critical path and the vast majority of students answered part c(i) correctly. Many students, however, had difficulty correctly calculating the slack time for task J (although a smaller proportion of the cohort than for a similar question in last year's examination).

The most common errors in part (d) involved leaving the month off the date in part (i) and not answering in context in part (ii). In part (e) many unsuccessful students either left the question blank, had the dummy link running from the beginning of task G instead of the end, or failed to show direction by leaving off the arrow.

Marking for part (f) was followed through from each student's response to part (e) and most could carry out the forward scan correctly for their network.

A great many of the students who answered parts (e) and (f) correctly also identified the second statement in part (g) as the most complete, as it includes information about both the completion time and the critical path. Those students who gave the most common incorrect answers for parts (e) and/or (f) rendered two of the statements in (g) untrue and hence the only acceptable answer for them for part (g) was ticking the first statement.

### Question 7 – Loans

This question proved quite challenging for many students as it incorporated both sinking fund and offset account calculations. Despite this complexity many students handled it well.

The comments made earlier about simple interest-type calculations and the use of positive and negative signs in TVM calculations apply equally in this question.

Most students were able to confirm the payment figure in part (b) but were not always successful in finding the total paid for the loan. The calculations are quite straight forward but care must be taken in choosing the correct number of payments for each part as they are different for the interest only loan and the sinking fund (as is often the case in this scenario).

Many students handled the first parts of the offset section of the question d(i) and (ii) adequately but struggled a little more with parts (iii) and (iv). The most common error in part (iv) was to forget the first two years of payments. Successful students in part (e) grasped that the best option was the cheapest, using figures from previous parts of the question to support their choice.

### Question 8 – Critical Path Analysis

Completing the precedence table in part (a) of this question proved an easy task for most students. Some listed all the preceding tasks rather than just those immediately preceding, for which there was no penalty applied in marking.

Adding the missing links to the graph proved more of a challenge with a significant number of students leaving the question blank. It is unclear whether these students did not know what to do or missed that there was a question to answer.

It is worth stressing here that there have always been questions in the examination which require students to add information to existing graphs, diagrams or tables and they should be made aware of the cues to look for so as not to lose marks unnecessarily by missing questions. Incorporating such questions into classroom tests and using past examination papers for practice are both good strategies for reducing the likelihood of this happening in the examination.

The most common error amongst those who answered the question in part b) was not having the link for task C connected to the finish node. Many of the students who drew the graph correctly in (b) also correctly identified the second statement in part (c) as the one which was false. Those who answered (b) incorrectly often rendered more than one of the statements false, in which case they were awarded the mark for part (c) if the statement they selected was indeed false for their particular network.

### Question 9 – Exponential regression

Each year since the introduction of exponential regression in the General Mathematics course the questions have been set to gradually introduce more complexity until this year when the full breadth and depth of the topic has been examined. Over this time the scripts have shown a steady improvement in student understanding and competence in their responses to the exponential regression question.

Many students found Question 9 challenging but the majority made some headway, particularly with the more standard parts of the question. In parts (a) and (b) they were asked questions leading to a rejection of a linear model for the data. The vast majority of students correctly chose graph 'D' for the line of best fit with the most common incorrect answer being graph 'B'. This response shows that most students are aware of what a linear model through given data should look like but some did not appreciate the subtleties of the position of the y-intercept. It is advisable that when asked this type of question students should produce the graph on their calculator and pay attention to such details.

Part (b) specifically required students to restrict their reasoning to the residual plot. Successful students noted both the shape of the plot and the size of the residuals, whereas less successful students referred to the shape of the scatterplot or the value of the correlation coefficient.

A large number of students were able to construct an appropriate exponential model for the data, however quite a few of those were penalised for not using the correct variables in part c(i). It should be noted here that severe rounding at this point should be avoided as the exponential model is sensitive to small variations that can have a large impact on subsequent answers. Students should be advised to keep two or three decimal places in their parameters for these models.

There were a significant number of students who could not interpret the values of the parameters 'a' and/or 'b' in context for their model, even though the number of such students is less than in previous years. Incorrect answers for c(iii) such as "the number of cases per day increase by 1.125 (or 125%)", for example, showed the beginnings of understanding but not enough to gain the mark.

Most students who created an exponential model in part c(i) were able to successfully use it to calculate the required values in parts d(i) and d(ii), with fewer being successful in d(iii). The openness of the question in d(iii) seems to have daunted some students which suggests that they may have been unfamiliar with the property of the exponential model which gives a constant doubling time (or halving time in the case of decay). As this is often a very useful concept in the contexts in which exponential growth occurs (e.g. population growth, radioactive decay, spread of disease, etc.) it is something that would be worthwhile teachers exploring with their students in the classroom.

The calculation involved in answering the question was not out of the ordinary. Students simply needed to double a value of 'C' for which they already knew the value of 'd', find the new 'd' and subtract (for example the most obvious choice would be to use  $C = 4.04$  when  $d = 0$ ,  $C = 8.08 \Rightarrow d = 5.9$ , hence about 6 days to double but they could also have used their answer to d(ii) when  $C = 10\,000$ ).

[A note here: It could be useful for teachers to instruct their students in the use of the equation solver in their calculator for answering questions like this one (since algebraic methods are not a requirement of the course). The alternative of using G-solve in the graph menu is quite a lot slower especially if the view window has to be adjusted.]

A very few students misinterpreted the question in part d(iii) to mean “When will the number of cases double in one day?” rather than how long does it take for the value of the variable ‘C’ to double. If these students explained their interpretation of the question clearly and justified the fact that this was impossible due to the constant daily percentage increase they were given credit for their understanding of this aspect of the mathematics of the model.

In the final part of the question the most successful students contrasted the infinite, ever increasing growth of the exponential model with limiting factors in the context such as finite population and measures put in place to curb the spread of the disease.