**Stage 2 General Mathematics**

**Assessment Type 2: Mathematical Investigation**

**Statistical Models – Rates of Cooling**

**Description**

The following data represents the temperature above room temperature of a cup of flat white coffee as it cools.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *t* (mins) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| *A* (°C) | 71 | 61 | 51.5 | 45 | 39 | 35 | 31 |

Where: *A* is the number of degrees above room temperature, and

*t* is the time in minutes that have passed since the coffee was poured.

In this task you will develop formulae to model the relationship between the number of degrees above room temperature of a cup of coffee and the time in minutes that have passed since the coffee was poured.

**Section 1**

1. Examine the data in the table above and predict, using only your ‘intuition’, what the temperature difference will be 2.5 minutes, 13 minutes, 35 minutes and 60 minutes after the coffee is poured. Predict how long it might take for the coffee to reach 5°C above room temperature.
2. On a large set of axes accurately plot the graph of *A* against *t*. Describe the shape of the graph and any of its significant features. Discuss whether it make sense to connect the dots with a continuous curve.
3. Use your graph to estimate the temperature of the coffee above room temperature at the four different times given in part 1 above. Compare your predictions from part 1 with these answers.
4. Enter the tabled data into your graphics calculator and fit an appropriate model to it. Give the algebraic formula for this model. Discuss your reasons for choosing the particular model you used with reference to the features and behaviour of its graph.
5. Use your model to predict the temperature of the coffee above room temperature for the same four times that were used in parts 1 and 3, as well as the time it would take for the coffee to reach 5°C above room temperature
6. Comment on similarities/differences in the temperature of the coffee as predicted using the three methods in parts 1, 3 and 5. Which values would you consider the most and least accurate predictions? Give clear reasons for your answers. Compare the two predictions made for the time it would take for the coffee to cool to 5°C above room temperature and comment on any difference between these two estimates.

**Section 2**

1. Consider what other factors might affect the rate of cooling of the coffee (e.g. size of the cup, what the cup is made of, initial temperature of the empty cup before the coffee is poured, ambient temperature of the room, type of coffee etc.). **Choose one** factor to investigate and describe in detail how you expect it might affect the graph of cooling of the coffee over time.
2. Design an experiment where two variations of your chosen factor are used and all other parameters are kept consistent (e.g. you could make a cappuccino coffee and then a long black coffee keeping everything else about the experiment the same). Through **direct measurement** produce a table similar to the table above for each of these two new situations.
3. Analyse your data using appropriate graphs and algebraic models. Compare any similarities and differences between the three models investigated. Conclude your investigation by summarising the important points discovered. Include discussion of the assumptions made in the investigation and the reasonableness of the models used.

**The investigation report should be a maximum of 12 single-sided A4 pages if written, or the equivalent in multimodal form.**

**Report Format**

The report may take a variety of forms, but would usually include the following:

* an outline of the problem and context
* the method required to find a solution, in terms of the mathematical model or strategy used
* the application of the mathematical model or strategy, including
* relevant data and/or information
* mathematical calculations and results using appropriate representations
* discussion and interpretation of results, including consideration of the reasonableness and limitations of the results
* the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

Performance Standards for Stage 2 General Mathematics

| - | Concepts and Techniques | Reasoning and Communication |
| --- | --- | --- |
| A | Comprehensive knowledge and understanding of concepts and relationships.  Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.  Successful development and application of mathematical models to find concise and accurate solutions.  Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | Comprehensive interpretation of mathematical results in the context of the problem.  Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.  Proficient and accurate use of appropriate mathematical notation, representations, and terminology.  Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.  Formation and testing of appropriate predictions, using sound mathematical evidence. |
| B | Some depth of knowledge and understanding of concepts and relationships.  Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.  Attempted development and successful application of mathematical models to find mostly accurate solutions.  Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems. | Mostly appropriate interpretation of mathematical results in the context of the problem.  Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.  Mostly accurate use of appropriate mathematical notation, representations, and terminology.  Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.  Formation and testing of mostly appropriate predictions, using some mathematical evidence. |
| C | Generally competent knowledge and understanding of concepts and relationships.  Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in different contexts.  Application of mathematical models to find generally accurate solutions.  Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | Generally appropriate interpretation of mathematical results in the context of the problem.  Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.  Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.  Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.  Formation of an appropriate prediction and some attempt to test it using mathematical evidence. |
| D | Basic knowledge and some understanding of concepts and relationships.  Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context.  Some application of mathematical models to find some accurate or partially accurate solutions.  Some appropriate use of electronic technology to find some accurate solutions to routine problems. | Some interpretation of mathematical results.  Drawing some conclusions from mathematical results, with some awareness of their reasonableness.  Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.  Some communication of mathematical ideas, with attempted reasoning and/or arguments.  Attempted formation of a prediction with limited attempt to test it using mathematical evidence. |
| E | Limited knowledge or understanding of concepts and relationships.  Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems.  Attempted application of mathematical models, with limited accuracy.  Attempted use of electronic technology, with limited accuracy in solving routine problems. | Limited interpretation of mathematical results.  Limited understanding of the meaning of mathematical results, their reasonableness or limitations.  Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.  Attempted communication of mathematical ideas, with limited reasoning.  Limited attempt to form or test a prediction. |