2021 Specialist Mathematics Subject Assessment Advice

Overview

Subject assessment advice, based on the 2021 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

School Assessment

Teachers can improve the moderation process and the online process by:

* thoroughly checking that all grades entered in schools online are correct. Errors in entered grades usually cannot be fixed through the moderation process, particularly if the error means a change in the rank order of results
* ensuring the uploaded tasks are legible, all facing up (and all the same way), and remove blank pages, student notes and formula pages
* ensuring the uploaded tasks also have pages the same size and in colour so teacher marking, and comments are clear
* ensuring the uploaded student SATs have been clearly marked showing which mathematical calculations are fully or partially correct and which are incorrect is a requirement of moderation. Showing marks and totals is also helpful
* for investigations, comments and clearly marked mathematical calculations are a requirement for the moderation process
* uploading the SATs as a single scanned file
* preferably providing a summary of student results in each of the SATs at the start of the uploaded SAT’s file
* ensuring uploaded investigations are the final work and not the draft. However, a draft can be assessed and uploaded if a student does not submit a final response
* using the same tasks where possible when combining with another school or schools to ensure standards are equitable. When combining classes across schools, teachers should be involved in moderation activities prior to up-loading materials for the actual moderation to ensure that the rank order of the students within the combined assessment group is appropriate.

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete six skills and applications tasks under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring the SAT is at a good level that allows for both routine questions, and questions that include enough complexity to allow achievement at the higher-grade bands
* designing some complex questions to allow students to progress one step at a time through a process, using the ‘Show that …’ style of question
* *s*tructuring questions with multiple parts that begin with ‘access’ points to elicit C grade evidence and subsequently increase in complexity, with the potential to elicit A grade evidence
* providing a marks scheme and working space reflective of the cognitive demand of the question
* providing students with appropriate feedback, including marks, to help them improve their work
* assessing conjecture and proof through this assessment type as they can be difficult to assess within a mathematical investigation
* please ensure that your LAP accurately indicates where RC5 is being assessed
* induction per se is not RC5, completing proofs using Mathematical Induction does not on its own achieve RC5. Students must be given the opportunity to form their OWN conjecture and to then prove it. Many teachers choose to assess this in the Induction SAT, for example by including a question of the form:

(a) Given the matrix  find (i)  (ii)  (iii) .

(b) On the basis of your answers to (a) make a conjecture about the matrix .

(c) Prove your conjecture using the principle of mathematical induction for all positive integers 

* referring closely to the key questions and key concepts in the subject outline when designing assessment tasks. While including material such as inequalities in Induction, Euler’s form or exponential form of a complex number and more complex integration by substitution can be considered an extension (as these concepts are outside the subject outline) it should not be to the detriment of including content required to be known, such as polar form for example, in the summative SAT
* setting a variety of SATs that include limited questions drawn directly from past examinations. Schools can use a mid-year examination to provide students with a formative experience of examination style assessment; however, it is recommended that examinations are not included among the Assessment Type 1 SATs
* preferably not marking crossed out work as work the student has crossed out will not be marked in the final examination
* preferably not awarding half marks as these are not awarded in the final examination and can inflate results and student expectations
* making students fully aware of the capabilities of their graphics calculator so they can make informed choices as to when and where to use it in completing SATs
* providing clear feedback on the appropriate use of mathematical notation, with particular attention needed for questions using vectors and integration which were identified as more common problem areas during moderation and examination marking.

The more successful responses commonly:

* provided clear and logical reasoning with correct mathematical notation
* displayed evidence that aligned with the question requirements (e.g. hence, exact)
* provided solutions that were efficient and demonstrated clear, logical and comprehensive understanding and interpretation of the question/problem.
* showed all algebraic working by providing all relevant steps, particularly for the ‘Show that …’ style of question
* stated any theorems and/or properties that were being applied to support answers
* used mathematically correct notation, particularly in questions using vectors and integration
* labelled axes and scales of graphs correctly and used the graphics calculator efficiently to draw both cartesian and parametric functions, paying attention to correctly labelling and representing asymptotes and correctly showing shape and behaviour of curves near asymptotes
* paid close attention to all details given in questions and the detail required in answering by showing conceptual thinking in their responses no matter how simple
* included appropriate steps in applying algorithms and did not miss vital steps, especially in ‘Show that…” questions where the answer is given in the question stem.

The less successful responses commonly:

* often did not attempt to answer questions, particularly more complex style questions
* displayed incorrect mathematical notation and/or limited communication of reasoning i.e. the solution did not successfully ‘flow’ to a logical end
* included many arithmetic and algebraic mistakes that complicated the nature of the solutions (e.g. an error causing the student to have polynomials that did not factorise easily)
* do not follow instructions that directed the student to use a particular method such as “implicit differentiation” or to use a previous result, either by instructing students to use specified parts of the question, or using the word hence.
* lacked the appropriate detail; where several marks have been allocated, all relevant conceptual steps are required
* did not communicate a good knowledge of the algorithms covered by the course, often evident through incorrect application of techniques to solve questions
* seemed unfamiliar with the capability of their graphics calculator.

Assessment Type 2: Mathematical Investigation

Students complete one mathematical investigation.

The subject of the investigation may be derived from one or more subtopics and should have minimal teacher direction. The task must afford students the opportunity to extend the investigation in an open-ended context. Students are encouraged to use a variety of mathematical (and other) software to enhance their investigation. It must be completed in a report format and must be no longer than 15 single-sided A4 pages with minimum font size 10. Appendices may be used to support the report but are not part of the assessment decision unless they are part of the 15 pages. Teachers should provide feedback where appropriate on the suitability of the direction a student may take with their investigation, especially where the investigation topic was chosen by the student and provide feedback on one draft. In draft feedback the teacher may direct the student’s attention to errors but must not explicitly correct these for the student.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring that the format of the investigation allows for an open-ended exploration of the problem where the student can show individual choices, refinements/improvements to models, justifying their rationale for these developments
* providing examples in the task sheet of what could be modelled, and structuring the investigation to encourage students to focus on different models, extend their interests and explore more complex models
* ensuring that the investigation is at an appropriate level of complexity, aligns well with the subject outline and does not limit student’s ability to achieve at the highest level
* not using question-and-answer style investigations, which limit student success ensuring the design of the task allows for and explicitly encourages the discussion of limitations and reasonableness of the modelling process:
* tasks that are designed to look at the generation of curves or shapes by altering values within formulae is not likely to result in individual work that is sufficiently open ended or allowing deep discussions concerning the reasonableness of solutions or limitations encountered
* examples of types of investigations that may limit student success are Bezier curves with a conjecture and proof that is not unique to the student, graphs of various rational functions, or graphs of other relations that may give altering shapes depending on values chosen. Some Bezier curve investigations may not reach a complex level of modelling and may limit the discussion of reasonableness and limitations
* the most recently updated wine glass investigation on the SACE website allows an open-ended approach after initially being directed. Students need to execute a significantly open-ended section to produce an investigation at a complex level.
* encouraging the correct use of notation and labelling of graphs, axes, scales etc.
* assisting students with unfamiliar software so that they can represent graphs etc. with appropriate information attached
* providing feedback through drafting and/or discussing the direction taken to ensure that what students plan to do will provide them with the opportunity to achieve at the higher-grade bands and the teacher may direct the student’s attention to errors but must not explicitly correct these for the student
* explaining clearly the 15-page (single-sided) limit and the corresponding appropriate use of appendices. For example, important or initial mathematical calculations should be provided in the main body of the report; however, repetitions of a calculation with variations to the figures can be provided in the appendices with the results clearly provided in the main body of the investigation (within a table or using some other concise manner of presentation)
* using investigations that do *not* have published solutions such as those provided by MASA to ensure that student work is unique and authentic.

The more successful responses commonly:

* provided detailed information about the investigation and the context in the real world
* read as a complete report, with sentences of explanation, not a series of dot-point-like ‘answers’ to an ‘assignment’
* included detailed explanations of all algebra, choices of values, and graphical work produced
* included graphical representations appropriately labelled to enhance the discussion within the investigation
* successfully developed a modelling situation with clear explanation of the decisions made throughout the mathematical investigation justified with reference to the real-life context and/or cited research and references as appropriate. This included mathematical calculations for each stage of development of the model that were commensurate with the cognitive demands of Stage 2 Specialist Mathematics
* demonstrated understanding of the reasonableness of the mathematical results and the limitations of the modelling process used, with attempts to improve, expand on, and develop based on these reflections, as appropriate
* used appropriate mathematical software to enhance the quality of the investigation
* used mathematical notation, representations, and terminology appropriately
* effectively communicated mathematical ideas and reasoning to develop logical arguments
* formatted their document so the mathematical notation flowed properly, and headings didn’t appear at the bottom of one page and the content at the top of the next page
* used appendices appropriately for repeated algebraic calculations to arrive at results.

The less successful responses commonly:

* had a limited introduction to the investigation, giving the reader little insight into the nature of the problem and the investigation to be undertaken
* had limited supporting evidence of how the models were derived e.g. trial and error, Geogebra, researched and adapted
* provided little evidence of effective use of technology. The investigation is an ideal assessment to implement a range of technologies to represent and solve problems leading to the development of the model
* read like a series of dot-point-answers as if the student just listed responses to an assignment or worksheet
* did not provide explanations or reasoning for the decisions made throughout the investigation
* made poor use of notation and often did not fully identify graphs
* included little or no labelling of diagrams
* followed the early direction given, but did not achieve much more, often failing to attempt the open-ended part of the investigation
* appeared to not have submitted their draft to the teacher for feedback.

External Assessment

Assessment Type 3: Examination

This year was the second year for a two-hour paper. The second book contained the longer style questions, worth 45 marks. As in past years the cohort who undertook the examination was made up of those students who knew their work and produced good to very good results, but there were a proportion of students who struggled to respond successfully.

Students found Book one with the shorter questions, worth 55 marks, more accessible than the longer questions in Book 2. Students found the 10 minutes of reading time useful for time to work on the problems.

General comments worth stressing:

* The ‘Show that …’ style of question requires students to show full working, displaying all steps of logic, for maximum marks. The style of solution here should be approaching one side of the given information and working towards developing the other side. The two sides should not be used together.
* An ‘exact answer’ means the answer should be in rational or irrational form without approximations to decimal values.
* Students need to be reminded that if the answer is stated in the question, marks are awarded for providing the working steps needed to reach this answer.
* Knowledge of, and the use of, a graphics calculator is assumed.
* Poor notation was often seen in student responses. Two areas of concern are the poor use of vector notation and integration notation. Students should also be mindful of using the variables in the question. For instance, if the question states,  then when finding the derivative,  the variable t should be used and not x.
* Students should recognise that earlier parts of a question are often relevant to the later parts of a longer question.
* Students should be aware of algebraic language. Some students did not use the brackets required to show a logical flow of their algebraic reasoning, which lead to errors in their mathematics.

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible, however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet, unless they are confident that no part of their response should be considered by the marker.

If a student crosses out a response and then decides that it was the correct (or the most correct) answer, then the student should indicate clearly to the marker which part of their response should be considered. This could be done by circling or highlighting all or part of the response that the student wants to be considered and write, for example, “please mark this work”. Students do not need to rewrite their answers in this case unless the crossing out has rendered the response unreadable.

It is advisable that students indicate in the space for an answer if they are also using the extra page for more working. For example, “see page x”. The work on the extra pages must be labelled clearly.

Specific comments for the questions within booklets 1 and 2 follow:

Booklet 1

Question 1

Approached well by the majority of students, which made for a positive start.

(a) The more successful responses for this question, worth only one mark, were those who chose to use their calculators to convert Cartesian form to polar form. Many students incorrectly stated  as the argument of the polar form.

(b) (i) Those students who connected part (a) to this question responded accurately.

(ii) Most students achieved one mark for this question recognising the double length of , but not many students indicated the correct position.

(c) The students who set up the triangle inequality using the triangle produced from their sketch in part (ii), and used the correct representation of the inequality, were successful.

Question 2

This question was attempted well by many.

(a) Recognition of the product rule to differentiate was mostly well done, provided students followed the rules of algebra and used brackets where appropriate.

(b) (i) Many students scored the mark for this basic algebraic manipulation.

(ii) Students who were careful with the required substitution of values into the work from part (a), and the rearrangement to find  in simplest form, performed well. Students should be conscious of cancelling fractions to their simplest form.

Question 3

(a) If students do not define  initially then using any working during the proof with  and  is not valid. The proposition must be defined to initiate the proof. The inductive step is crucial for the proof and hence showing working from the left-hand side of  as  and using the *k*th proposition to find the right-hand side of the proposition is the appropriate process. Some students incorrectly used both sides of the . It was encouraging that students were able to multiply matrices using assumed knowledge from Stage 1 Mathematics.

(b) Some good responses were seen in this part from students who used index laws correctly.

Question 4

(a) Students who used polynomial long division for this question were the most successful.

(b) Many students did not include the oblique asymptote of  which should be able to be drawn correctly at this level. Connecting part (a) reveals there is an oblique asymptote. Some students who did include the asymptote did not indicate that the graph cuts the oblique asymptote.

(c) This question well done.

(d) (i) There is a given result in this question, therefore students need to show more than a formula that is available of the formula sheet provided in the response.

(ii) This question requires an exact answer and being worth 3 marks, required integration techniques to be employed. Students should show their working and evaluate exactly the expressions.

Question 5

(a) Many students correctly identified the augmented matrix form in this question.

(b) Students were directed to state row operations, so those who approached this displaying their working were the most successful.

(c) (i) Many students recognised the values of *m* for which there was not a unique solution, but the question asked for a possible *m* value. Any single value other than  or  was acceptable.

(ii) There was a need to recognise that no planes are parallel in this scenario, so the best suited diagram is given in Figure D.

Question 6

(a) Most students attempted the integration by following the instructions to use the ‘by parts’ method. The most common loss of mark(s) was due to not clearly showing the development of the integral.

(b) (i), (ii) and (iii) The graphs were generally done well, but students must take note of the domain information. Many students included arrows on the graph instead of taking note of the domain.

(c) Not many students completed this successfully. Linking the ideas from the previous graphs was necessary to see the area required and the easiest approach to finding it. Many missed the area of a rectangle ( ) with the section of the area under the curve of the function 

Question 7

(a) This question was answered well.

(b) (i) Most students started off reasonably well and received 3 marks out of the 5 marks available. The greatest difficulty seen in responses was the use of the initial conditions to find the constant of integration.

(ii) The most successful students approached this question by using their calculator capabilities to solve an equation. Some students missed the second mark by not rounding the final answer correctly.

(iii) Mostly well done. Most students started the solution curve at the point (0, 20000) but some did not include the information from part (ii): the point (50, 5000). As well, displaying asymptotic behaviour to the *t* axis was required.

Booklet 2.

The three questions in this booklet are longer and worth 45 marks in total. The parts within the questions are connected.

Question 8

This question provided opportunities for many students to display their knowledge and gain marks.

(a) (i) and (ii). Part (i) was mostly well done although some students did not find the appropriate vectors for the required cross product calculation. Those students who use their calculator for the cross product answered efficiently. In part (ii) it is required for students to indicate they are using the normal to the plane from their work in part (i), as well as stating a point they are using, to develop the given equation. Some students did not give enough clear information indicating the use of the result from part (i).

(b) (i) and (ii). Well done by most. In part (i) different styles of finding evidence that AD is the diameter of the circle were seen by markers. In part (ii) most students found the radius with no issue.

(c) Most students were able to answer this question for the full marks. Students should state the vector  clearly in their working and state they are using the point *E*.

(d) It was pleasing to see so many students being successful in this question. Some used their calculator well to solve an equation to find the t value required. Clear substitution of this t value into the parametric equations was required for full marks.

(e) Students found this last part difficult, with many finding  rather than the arcBX. Some attempted to find angle BCX but used the incorrect vectors for the scalar product which led to an incorrect angle. Some students managed to see the need for using 

Question 9

(a) (i) – (iii). Part (i) Students needed to take care with entering the details into their calculators correctly. If a set of axes is given, then this should be used for the axes scaling in the calculator. Also, knowledge of how to enter the restrictions for the *t* variable is important. Some students incorrectly included arrows on the graph. Parts (ii) and (iii) were well done generally, but a reminder to students to follow instructions and label the point when required.

(b) (i) – (iii). In part (i) some students did not realise  but for some this was the only mark they gained due to poor knowledge of differentiation of trigonometric expressions. Part (ii) was well done. For part (iii) some students labelled the slope of the tangent as a point rather than a line with slope 

(c) (i) A reminder again that students need to show working when a result is given. Just stating a formula is not sufficient.

(ii) On the whole, students who used their calculators for this question were successful.

Question 10

(a) (i) and (ii). In part (i) if students write solutions in the form they must give the values of *k* used for generating the six solutions. Plotting the solutions in part (ii) was generally well done, but students must attempt to indicate the length and position of the solutions.

(b) (i) – (iii) In part (i) most students saw the connection from part (a) and added 1 to the solutions found earlier. Some students incorrectly subtracted 1 for the solutions. The lack of understanding of the translation of solutions was also evident in part (ii) when the new solutions had to be drawn. In part (iii) it seemed that some students incorrectly interpreted ‘real form’ to be Cartesian form.

(c) (i) Many students recognised the need for the conjugate form, . Utilising the sum and product of roots was the most successful method, although some students stated the sum of roots as  in their working.

(ii) Generally, students attempted binomial expansion using Pascal’s Triangle, but failed to be clear in their working by not using brackets around the powers of 

(d) Those students who used the direction of connecting earlier parts of the question were the most successful.