



General Mathematics

2021

Question booklet

- Questions 1 to 9 (90 marks)
- Answer **all** questions
- Write your answers in this question booklet
- You may write on page 22 if you need more space

Examination information

Materials

- Question booklet
- SACE registration number label

Instructions

- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used — complete the box below

Total time: 130 minutes

Total marks: 90

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<p>Attach your SACE registration number label here</p>	<p>Graphics calculator</p> <table border="0"><tr><td>1. Brand</td><td>_____</td></tr><tr><td>Model</td><td>_____</td></tr><tr><td>2. Brand</td><td>_____</td></tr><tr><td>Model</td><td>_____</td></tr></table>	1. Brand	_____	Model	_____	2. Brand	_____	Model	_____
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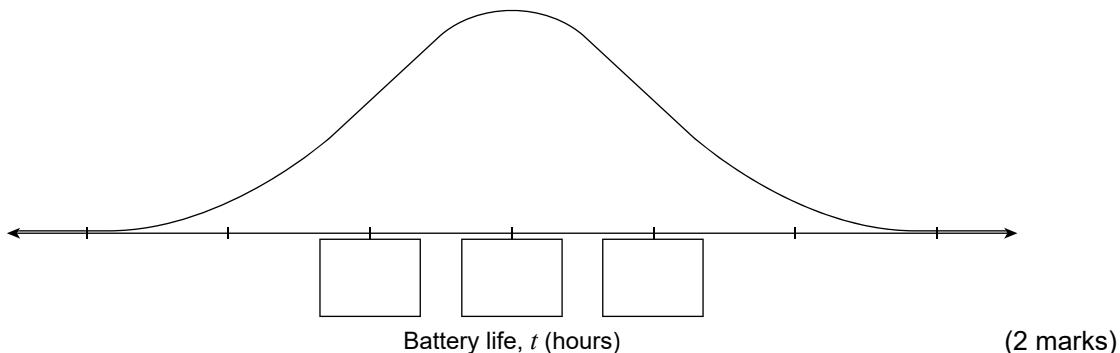
Government
of South Australia

Question 1 (10 marks)

Testing has been carried out on the length of time (t hours) that a new Brand X smartphone battery will work before the phone switches off because it needs charging. A large number of Brand X phone batteries were tested and t was found to be normally distributed with a mean life of 16.3 hours and a standard deviation of 2.4 hours. All batteries were fully charged before being tested.

- (a) The graph below represents the normal distribution for Brand X smartphone batteries.

Write a number in each box to provide a partial scale for the distribution.



- (b) (i) Calculate the percentage of Brand X batteries that could be expected to last less than 15 hours.

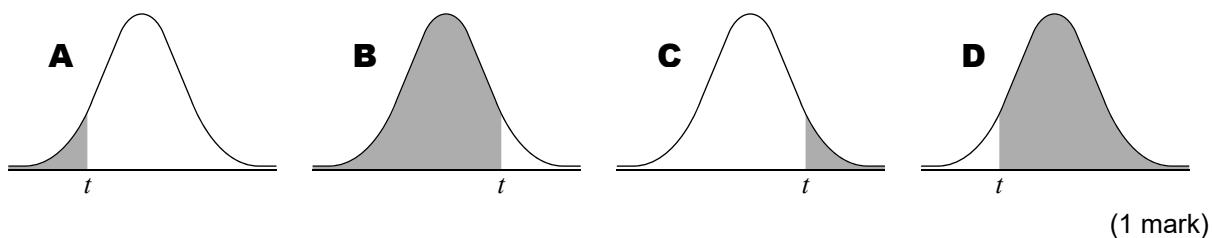
(1 mark)

- (ii) If 834 Brand X batteries were tested, how many could be expected to last less than 15 hours?

(2 marks)

- (c) Brand X advertisers claim that 90% of their smartphone batteries will last at least t hours.

- (i) **Circle** the letter of the graph below that represents the claim.



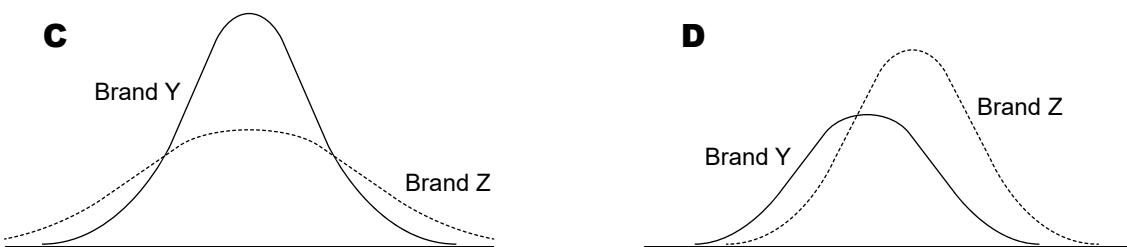
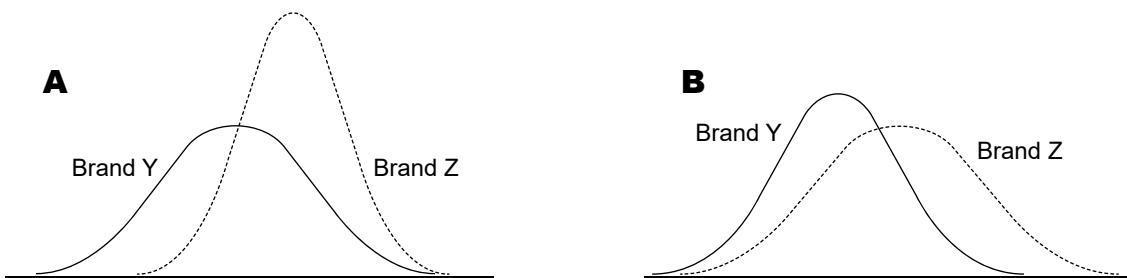
- (ii) Calculate the value of t . Give your answer correct to at least one decimal place.

(1 mark)

The manufacturers of Brand X batteries extended their testing to include two rival brands of smartphone battery, Brand Y and Brand Z. It was found that t was normally distributed for the two rival brands.

- Brand Y batteries: mean life of 18 hours, standard deviation of 3 hours.
 - Brand Z batteries: mean life of 21 hours, standard deviation of 4 hours.

(d) (i) **Circle** the letter of the graph below that represents the distributions of battery life for Brand Y and Brand Z batteries.



(1 mark)

(ii) ‘Brand Z batteries are more dependable than Brand Y batteries.’

Describe two features from the graph that you selected in part (d)(i) that support this statement.

(2 marks)

Question 2 (11 marks)

As a newly graduated accountant, Eilidh started a job earning \$60 430 per annum. She decided to contribute 3% of her income into a superannuation fund which earned interest at 5.85% per annum, compounded quarterly. Her employer contributed a further 10% into the same account.

- (a) Show that payments of approximately \$1960 per quarter were contributed to Eilidh's superannuation fund.

(1 mark)

- (b) Hence calculate the expected balance in the account after 43 years of Eilidh working in this job.

(2 marks)

- (c) State two different reasons why the balance after 43 years might end up being higher than the balance that you calculated in part (b).

(2 marks)

After some years of working, Eilidh achieved a balance of \$120 000 in her superannuation account. At this time she decided to take a career break for 7 years. During this break, no payments were made into her superannuation account.

- (d) What interest rate, compounding quarterly, would Eilidh have needed on her superannuation account if she was to grow the balance to \$200 000 over the 7 years?

(2 marks)

- (e) Eilidh retires with a balance of \$900 000 and places it into an account earning 3.65% per annum, compounding weekly.

- (i) If she wants to still have \$900 000 in the account after 10 years of retirement, how much can Eilidh withdraw per week?

(2 marks)

- (ii) At retirement, Eilidh decides to withdraw \$1250 per week to live on.

Calculate how many years it will be before the account balance reduces to \$0.

(2 marks)

Question 3 (14 marks)

Four items have been donated to a charity auction. A condition of the auction is that any buyer can purchase only one item. The organisers of the auction will decide which buyer is successful for each item so that the maximum amount of money is raised for the charity.

Four buyers place their bids on each of the items in the auction as shown in the table below (in hundreds of dollars).

	Bids ($\times \$100$)			
	Item 1	Item 2	Item 3	Item 4
Buyer 1	10	13	20	22
Buyer 2	7	13	21	17
Buyer 3	6	12	23	19
Buyer 4	9	16	21	20

- (a) Based on the information in the table, which buyer has made the highest bid on Item 2? Tick the appropriate box to indicate your answer.

Buyer 1

Buyer 2

Buyer 3

Buyer 4

(1 mark)

- (b) (i) Explain what has been done to the original array to produce the one below and why it is necessary to do this.

13	10	3	1
16	10	2	6
17	11	0	4
14	7	2	3

(2 marks)

- (ii) Carry out the Hungarian algorithm on the array from part (b)(i) given below until a solution is reached. Show the result of each step clearly.

13	10	3	1
16	10	2	6
17	11	0	4
14	7	2	3

(5 marks)

- (iii) Fill in the spaces below to complete the interpretation of your solution from part (b)(ii).

Buyer 1 will get Item _____

Buyer 2 will get Item _____

Buyer 3 will get Item

Buyer 4 will get Item

The charity will raise \$ [REDACTED] from the auction.

(3 marks)

Question 3 continues on page 8.

Before the day of the auction, Item 4 was withdrawn from sale and the organisers decided to allow Buyer 2, who is a patron of the charity, the chance to purchase two items. The buyers were then allowed to change their original bids.

As a result of these changes, a new initial array has been produced:

- The bids made by Buyer 2 are repeated in row 3.
- Two dummy columns have been added.

		Bids ($\times \$100$)				
		Item 1	Item 2	Item 3	Dummy	Dummy
Buyer 1		17.5	9	24	0	0
Buyer 2		17.5	7.5	23	0	0
Buyer 2		17.5	7.5	23	0	0
Buyer 3		13.5	8	20	0	0
Buyer 4		14	7.5	19	0	0

- (c) The Hungarian algorithm has been performed on this array, with the result shown below. It is ready to be used to assign the items to buyers, and multiple solutions are possible.

1	0	0	1	1
0	0.5	0	0	0
0	0.5	0	0	0
4	0	3	0	0
3.5	0.5	4	0	0

- (i) Complete the statement below.

In **all** possible solutions, Buyer _____ will get Item _____.

(1 mark)

- (ii) State another outcome which is the same in **all** possible solutions.

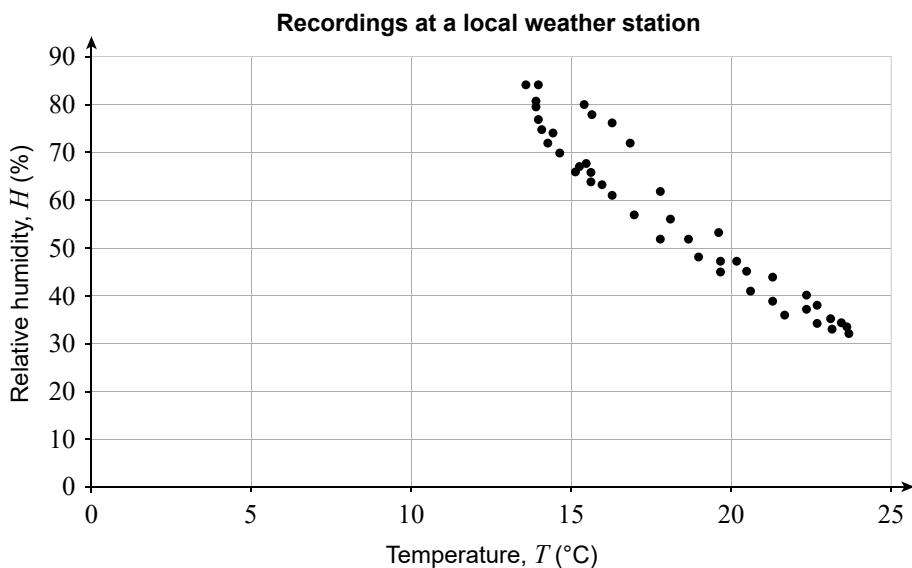
(1 mark)

- (iii) Calculate the difference in the total amount of funds raised for the charity as a result of the changes made above.

(1 mark)

Question 4 (9 marks)

The scatter plot below shows the temperature, T ($^{\circ}\text{C}$), plotted against the relative humidity, H (%), over 24 hours at a local weather station.



A linear model has been fitted to the data. The least squares regression equation and the value of r^2 for the model are:

$$H = -4.68T + 141.85 \quad r^2 = 0.929$$

- (a) Considering the information above, which statement below best fits the linear model? Tick the appropriate box to indicate your answer.

A small, empty square box with a black border, likely used for drawing or marking.

There is a moderate linear correlation between temperature and relative humidity.

1

The y -intercept for the model occurs at -4.68°C .

1

The r value for the least squares regression will be negative.

(1 mark)

- (b) Interpret the slope of this least squares regression line in the context of the problem.

(2 marks)

Question 4 continues on page 10.

The following data was recorded over the same 24-hour period at another local weather station.

Temperature, T ($^{\circ}\text{C}$)	15.6	16.3	17.8	19.7	21.3	22.7	23.7	18.1	18.9	15.7	12.1
Relative humidity, H (%)	66	61	52	46	39	34	33	56	75	78	84

- (c) Using a linear model, calculate the r^2 value for the relationship between the variables in the data above.

(1 mark)

- (d) Which of the following three points from the data is *most* likely to be an outlier? Tick the appropriate box to indicate your answer.

(18.9, 75)

(15.7, 78)

(12.1, 84)

(1 mark)

- (e) The outlier has been found to be the result of a recording error.

Remove the point chosen in your answer to part (d) from the data and calculate the new least squares regression equation. Write your answer in the space below.

(1 mark)

- (f) Use your equation from part (e) to predict:

- (i) the temperature when the relative humidity is 100%.

(1 mark)

- (ii) the relative humidity when the temperature is 32°C.

(1 mark)

- (g) The predictions made in part (f)(i) and part (f)(ii) are both extrapolations.

Complete the statement below.

The prediction made in part _____ would be the less reliable of the two predictions because

(1 mark)

Question 5 (8 marks)

Ryan wishes to invest some money for 4 years. The local credit union has given him three options:

- Option A — 2.07% per annum, compounding fortnightly
 - Option B — 2.10% per annum, compounding annually
 - Option C — 2.15% flat rate.

(a) (i) Calculate the effective rate of interest for Option A and for Option C.

(2 marks)

(ii) Hence tick a box to indicate which of the three options (A, B, or C) Ryan should select, and provide a reason for your choice.

Option A

Option B

Option C

(1 mark)

Ryan has \$5000 in savings and he decides to invest it in another account paying 3.28% per annum, compounding weekly. He also deposits \$45 each week into the account from the time he opens it.

(b) (i) Show that Ryan will have approximately \$15 700 in the account after 4 years.

(2 marks)

(ii) Calculate the total amount of interest that Ryan will earn in the account over the 4 years.

(1 mark)

- (iii) If Ryan doubled his weekly deposit from the time he opened the account, state why the interest calculated in part (b)(ii) would not double.

(1 mark)

Ryan is planning to travel in 4 years' time using the money from his savings account. The holiday he has planned would *currently* cost him \$14 000.

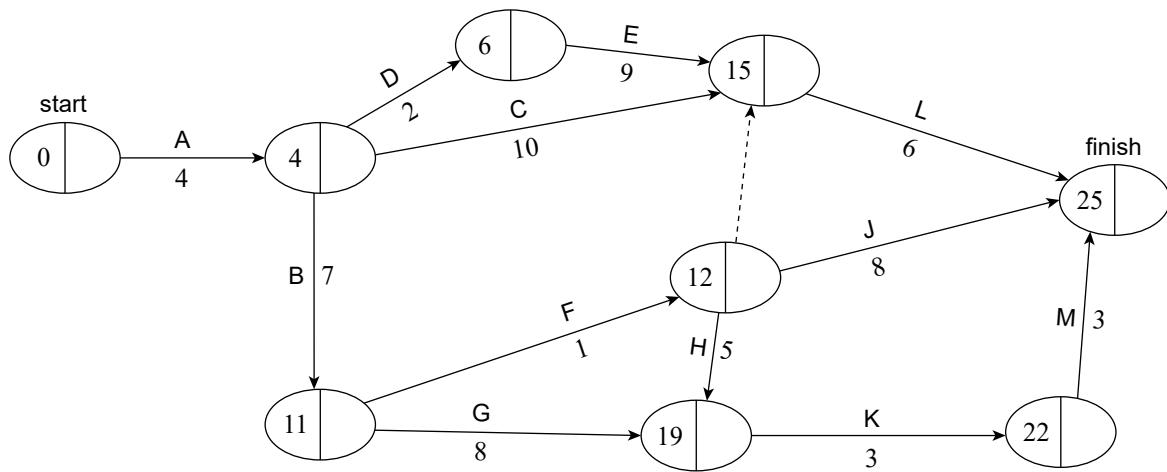
- (c) Assuming that the cost of the holiday increases with inflation, what is the *maximum* average annual inflation rate that could occur over the 4 years so that the cost of Ryan's trip rises to no more than \$15 700?

Tick the appropriate box to indicate your answer.

2.89% 2.90% 2.91% 2.92% (1 mark)

Question 6 (9 marks)

Thanh wants to landscape her garden and has identified the tasks that need to be completed. The times needed (in days) to complete each task, and the order for completion, are shown in the network diagram below.



- (a) State the tasks *immediately* preceding task L.

(1 mark)

- (b) On the network diagram above, carry out the backward scan.

(1 mark)

- (c) (i) State the critical path.

(1 mark)

- (ii) Calculate the slack time for task J.

(1 mark)

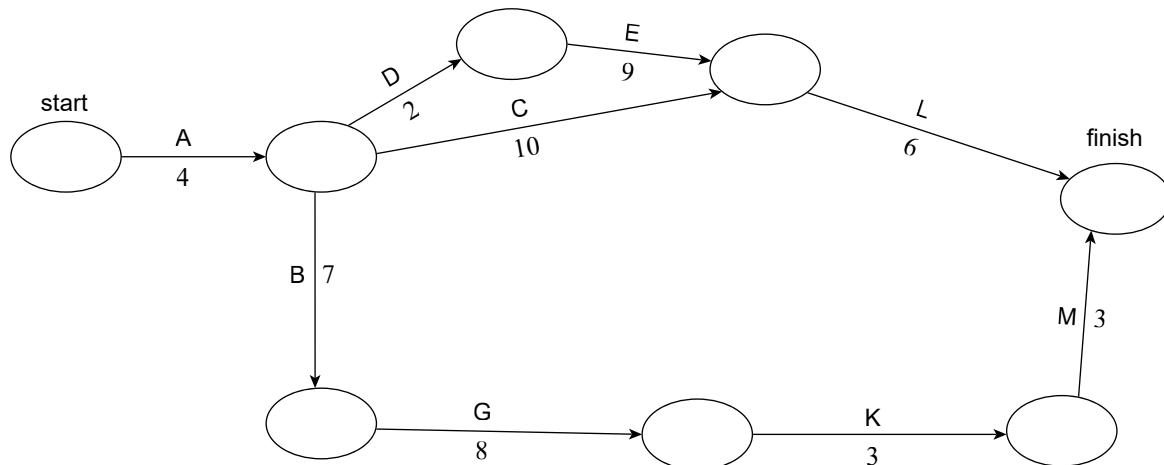
- (d) (i) If Thanh starts landscaping on 1 December, on what date will her garden be finished?

(1 mark)

- (ii) Apart from affordability, in the context of the problem state *one* assumption made when giving your answer to part (d)(i).

(1 mark)

Due to problems with her budget, Thanh has had to remove tasks F, H, and J from the plans for her garden. This is represented in the new network diagram below.



- (e) The changes will mean that task G is now a prerequisite task for task L, as well as for task K.

Represent this change on the network diagram above.

(1 mark)

- (f) Carry out a **forward scan** on the network diagram above. A backward scan is **not** required.

(1 mark)

- (g) All the statements below are **correct**. By ticking one box, select the statement which is the *most complete* interpretation of the effect that the changes have made.

1

Only non-critical tasks were removed, so the completion time is still the same.

1

The completion time is unchanged, but the number of critical paths has increased.

1

The earliest starting time for task L has increased, which has changed the critical path.

(1 mark)

Question 7 (11 marks)

The management of a marina investigates borrowing \$450 000 to refurbish moorings and upgrade the security system.

One option available is to take out an interest-only loan for 8 years and set up a sinking fund to pay back the principal at the end of the loan.

- (a) The interest on the loan will be charged at a flat rate of 7.39% per annum.

Calculate the monthly interest payment.

(1 mark)

- (b) The sinking fund to be established by the management earns interest at 3.25% per annum, compounding fortnightly.

Show that the payments required would be approximately \$1895 each fortnight.

(2 marks)

- (c) Calculate the total repayments to be made over 8 years by the management for the interest-only loan and sinking fund option.

(1 mark)

As an alternative option, the management considers taking out a reducing-balance loan with an offset account for 8 years.

After 2 years of payments, the balance owing on this loan would be \$351 700. At this time the management would be able to deposit \$15 000 into the empty offset account.

- (d) (i) State the new balance that interest will be calculated on as soon as the \$15 000 is deposited into the offset account.

(1 mark)

The interest charged on this loan is 4.35% per annum, compounding monthly, and the monthly repayments would be \$5558.74.

- (ii) Show that the balance of the loan on which interest is being charged 6 months after the deposit is made into the offset account would be approximately \$310 400.

(2 marks)

Six months after the deposit is made into the offset account, the management will need to withdraw the \$15 000.

- (iii) Calculate the remaining number of months needed to pay out the loan.

(1 mark)

- (iv) Calculate the total amount required to repay the reducing-balance loan if the offset account contains \$15 000 for the 6-month period.

(2 marks)

- (e) The management investigated two options for borrowing the money:

- Option 1 — the interest-only loan and sinking fund
 - Option 2 — the reducing-balance loan and offset account.

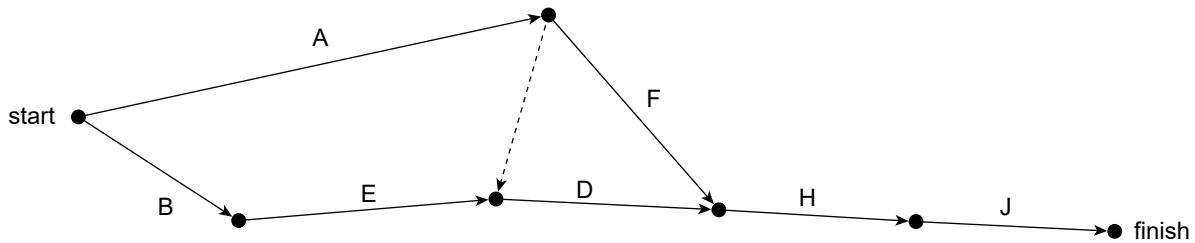
Complete the statement below.

The management should choose Option because

(1 mark)

Question 8 (6 marks)

The partial network diagram below represents the tasks required to sort mail in a post office.



- (a) Referring to the network diagram above, *complete* the precedence table below for tasks D, F, and H.

Task	Precedence
A	none
B	none
C	A
D	
E	B
F	
G	A, E
H	
J	G, H

(3 marks)

- (b) **Complete the network diagram above** by adding the links for tasks C and G. (2 marks)

- (c) Which *one* of the statements below is **false**? Tick the appropriate box to indicate your answer.

- Either task A or task E *must* lie on the critical path.
 Tasks A and B *cannot* both lie on the critical path.
 Tasks A, B, and E *must* all be complete before task D can start.
 There is no task which *must* lie on the critical path.

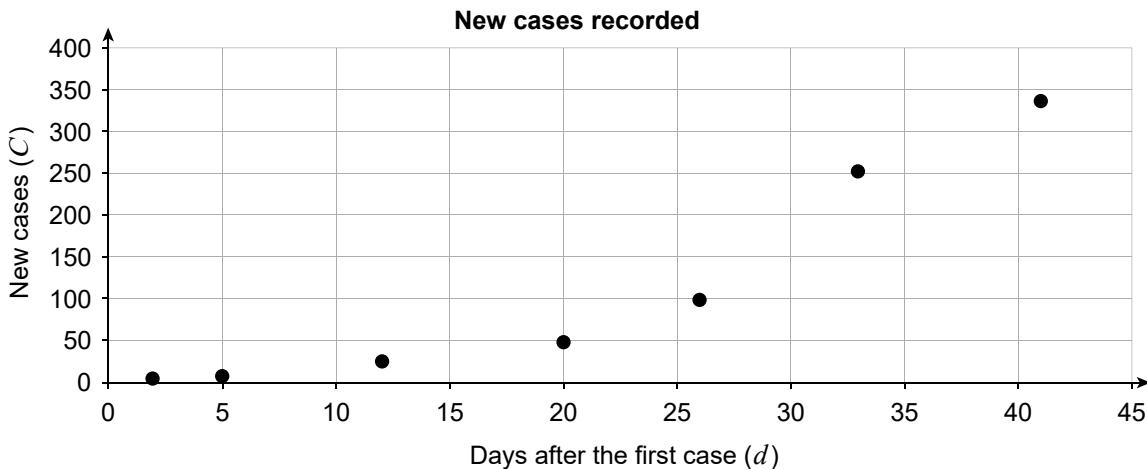
(1 mark)

Question 9 (12 marks)

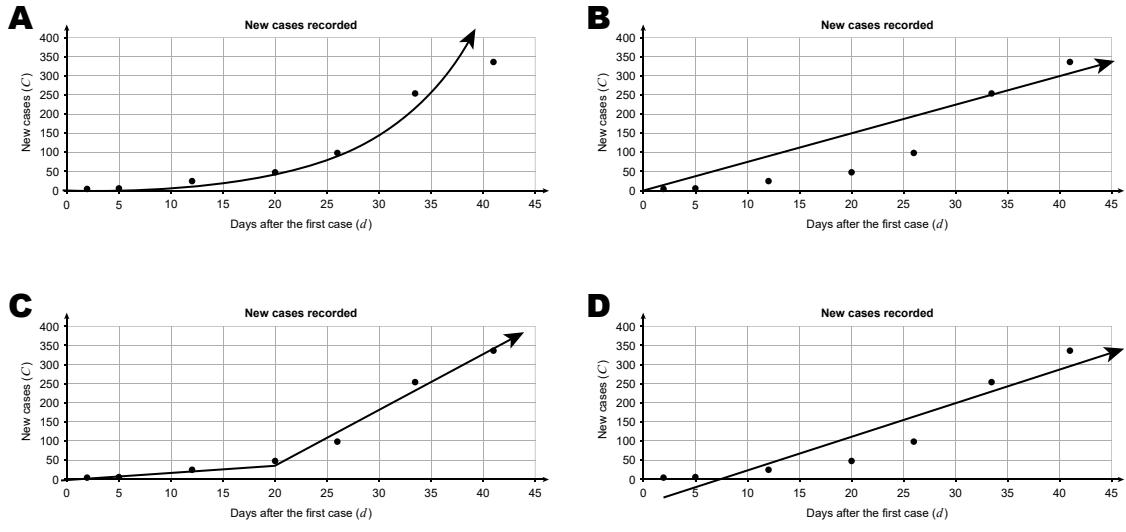
A health department identified an outbreak of a notifiable contagious disease. The data in the table below show the number of new cases on selected days after the first case was recorded.

<i>Days after the first case (d)</i>	2	5	12	20	26	33	41
<i>New cases recorded on the day (C)</i>	3	8	25	47	100	252	337

The scatter plot for the data is shown below.



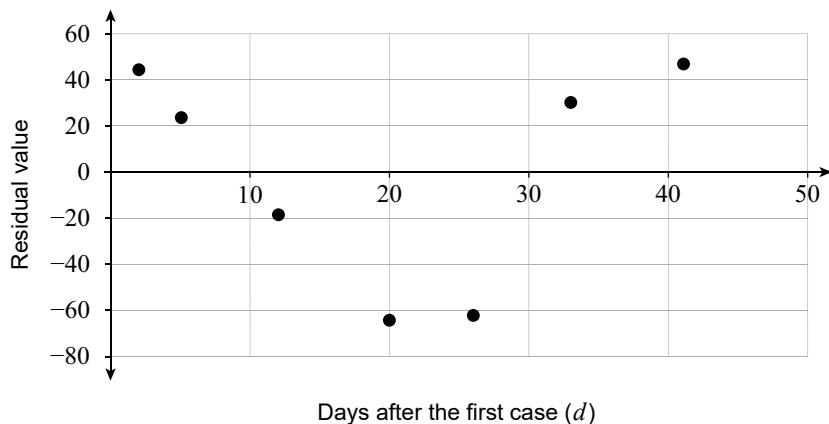
(a) **Circle** the letter of the graph below which shows the line of best fit for the *linear* model.



(1 mark)

Question 9 continues on page 20.

The diagram below shows the residual plot for the linear model on the previous page.



- (b) Using the residual plot above, give *two* reasons why it would ***not*** be appropriate to apply a linear model to this data.

(2 marks)

- (c) (i) Determine an exponential model for the data in the form $y=a.b^x$, where a and b are constants and the variables are appropriate for the context.

(2 marks)

- (ii) Interpret the value of a in the context of this problem.

(1 mark)

- (iii) Interpret the value of b in the context of this problem.

(1 mark)

(d) Using the exponential model that you found in part (c)(i):

- (i) Calculate the number of new cases of the disease that would be recorded on the 60th day after the outbreak started.

(1 mark)

- (ii) Calculate how many days after the start of the outbreak it would take for the number of new cases recorded in a day to reach 10 000.

(1 mark)

- (iii) Calculate how long it takes for the number of new cases recorded in a day to double.

(1 mark)

- (e) Explain why an exponential model for a contagious disease must eventually become an unreasonable model for making predictions as the disease progresses.

(2 marks)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 3(b)(ii) continued).

A large grid of 20 columns and 25 rows, intended for writing additional answers. The grid is composed of thin, light gray lines forming small squares.