**STAGE 2 GENERAL MATHEMATICS**

**Assessment Type 2: Mathematical Investigation**

**Modelling with Transition Matrices**

**Introduction**

There are currently 2 similar fruit and vegetable shops in one area – shop A and shop B. The initial market share of shop A is **X**%.

A survey of customers carrying out their weekly grocery shopping at each shop revealed that 76% of customers buying their weekly groceries at shop A intended to do their grocery shopping at shop A the following week, and 61% of customers buying their weekly groceries at shop B intended to do their weekly grocery shopping at shop B the following week.

**Mathematical Investigations**

A i. Decide on an initial market share value (X%) for shop A. Using this information and the shopping trend information provided above, make a prediction about the market share of shop A *in the long run*. Give reasons for the prediction you have made.

ii. Using matrix methods determine the market share of the two shops *in the long run* if the transition conditions remain the same. Does your result support your prediction?

B i. Make a prediction about what will happen to the market share of shop A *in the long run* if its initial market share changes (e.g. the X% increases or decreases). Give reasons for the prediction you have made.

ii. Using matrix methods investigate the impact on the long term trends of varying the initial market share for the two shops. Does your result support your prediction? Does the time it takes to reach a steady state vary with changes to the initial state matrix?

C Using matrix methods investigate the effect of one of the shops mounting a strong advertising campaign.

D A third cheaper shop (shop C) enters the market a few months later, and achieves an initial market share of no more than 15%. A new survey reveals the following trends:

* + - 12% of shop A customers will shop at shop B the following week
    - **M**% of shop A customers will shop at shop C the following week
    - 40% of shop B customers will shop at shop B the following week
    - **N**% of shop B customers will shop at shop C the following week
    - 14% of shop C customers will shop at shop A the following week
    - 7% of shop C customers will shop at shop B the following week.

1. Decide on the initial market share of the three shops, giving reasons for the values you have chosen.
2. Decide on the values of **M** and **N** for the transition behaviour of customers moving from shop A and shop B to the new shop C. Give reasons for the values you have chosen.
3. Using matrix methods consider:

* the market trends after different periods of time (weeks) and also the length of time at which the steady state is achieved with different initial market share values being used
* the effect of changes to the scenario, e.g. one shop extending trading hours to encourage customers to change shopping habits. You will need to decide on the new transition behaviour for the customers of the three shops with each change in scenario investigated. (Note: You should consider at least two different ‘change’ scenarios)

**Analysis/Discussion**

Critically analyse your results, considering:

* the information your calculations have provided
* possible implications of the investigation.

**Conclusion:**

The conclusions should include a summary of results, comments on the appropriateness of the model used, the reasonableness of the results, and any assumptions and limitations of the investigation.

**Notes to teacher:**

The 2×2 transition matrix used in the initial investigations is considered routine in nature; however for the purpose of the initial investigations it is appropriate. To achieve a level of complexity 3×3 matrix systems or higher must be utilised.

**The investigation report should be a maximum of 12 single-sided A4 pages if written, or the equivalent in multimodal form.**

**Report Format**

The report may take a variety of forms, but would usually include the following:

* an outline of the problem and context
* the method required to find a solution, in terms of the mathematical model or strategy used
* the application of the mathematical model or strategy, including:
  + relevant data and/or information
  + mathematical calculations and results using appropriate representations
  + discussion and interpretation of results, including consideration of the reasonableness and limitations of the results
* the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

Performance Standards for Stage 2 General Mathematics

| - | Concepts and Techniques | Reasoning and Communication |
| --- | --- | --- |
| A | Comprehensive knowledge and understanding of concepts and relationships.  Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.  Successful development and application of mathematical models to find concise and accurate solutions.  Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | Comprehensive interpretation of mathematical results in the context of the problem.  Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.  Proficient and accurate use of appropriate mathematical notation, representations, and terminology.  Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.  Formation and testing of appropriate predictions, using sound mathematical evidence. |
| B | Some depth of knowledge and understanding of concepts and relationships.  Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.  Attempted development and successful application of mathematical models to find mostly accurate solutions.  Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems. | Mostly appropriate interpretation of mathematical results in the context of the problem.  Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.  Mostly accurate use of appropriate mathematical notation, representations, and terminology.  Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.  Formation and testing of mostly appropriate predictions, using some mathematical evidence. |
| C | Generally competent knowledge and understanding of concepts and relationships.  Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in different contexts.  Application of mathematical models to find generally accurate solutions.  Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | Generally appropriate interpretation of mathematical results in the context of the problem.  Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.  Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.  Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.  Formation of an appropriate prediction and some attempt to test it using mathematical evidence. |
| D | Basic knowledge and some understanding of concepts and relationships.  Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context.  Some application of mathematical models to find some accurate or partially accurate solutions.  Some appropriate use of electronic technology to find some accurate solutions to routine problems. | Some interpretation of mathematical results.  Drawing some conclusions from mathematical results, with some awareness of their reasonableness.  Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.  Some communication of mathematical ideas, with attempted reasoning and/or arguments.  Attempted formation of a prediction with limited attempt to test it using mathematical evidence. |
| E | Limited knowledge or understanding of concepts and relationships.  Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems.  Attempted application of mathematical models, with limited accuracy.  Attempted use of electronic technology, with limited accuracy in solving routine problems. | Limited interpretation of mathematical results.  Limited understanding of the meaning of mathematical results, their reasonableness or limitations.  Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.  Attempted communication of mathematical ideas, with limited reasoning.  Limited attempt to form or test a prediction. |