**STAGE 1 MATHEMATICS**

**PROGRAM 1 – SEMESTER 1**

This program is for a cohort of students intending to continue to Mathematical Methods at Stage 2. The following program describes the first semester of learning.

**SEMESTER ONE 18 WEEKS INCLUDING EXAM WEEKS**

* Topic 1 – Functions and Graphs (5 weeks)
* Topic 2 – Polynomials (5 weeks)
* Topic 3 – Trigonometry (6 weeks)

**Topic 1 – Functions And Graphs (5 Weeks)**

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| **Term****week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 1-1 | 1.1Lines and Linear Relationships | The equation of a straight line* Given two points
* Given the slope and a point
* Parallel to a line through a given point
* Perpendicular to a line through a given point

Features of the graph of a linear function of the form $y=mx+c$* Slope (*m*) as a rate of growth
* Y-intercept *(c*)

Determine the formula for a linear relationship given data or description of situation* Various problems are addressed from everyday situations such as simple interest and conversion graphs. Slope as a rate of growth and interpretation of intercepts are considered in context.

Calculation of points of intersection* Solve simultaneous equations algebraically and graphically
* Consideration given to situations involving coincident, perpendicular and parallel lines
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| 1-2 | 1.2Inverse Relationships | Exploring the mathematical relationship where one variable decreases as the other increasesConsider the graph of the basic hyperbola $y=\frac{1}{x}$* Define asymptote, both horizontal and vertical

Consider translations of the basic hyperbola i.e. $y=\frac{a}{x-c}$The use of technology is incorporated in the graphs above |  |
| 1-3 | 1.3Relations | Definition of a relation as a set of ordered pairs* Discuss various examples

Exploration of the circle as a relation * Development of the equation of a circle in centre radius form
* Development of the circle in expanded (general) form, demonstrate the use of completing the square to convert from general to centre radius form
 |  |
| 1-4 | 1.4Functions | Definition of a function as a set of ordered pairs whereby no two have the same $x$ value (it defines one variable in terms of one other)* The graph of a function
* Domain and range
* Function notation
* Dependent and independent variables
* Use of vertical line test to establish a function

Understanding the distinction between functions and relations |  |
| 1-5 |  | **Revision and SAT 1** | **SAT 1**Entire topicCalculator permitted |

**Topic 2 – Polynomials (5 weeks)**

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| **Term****Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 1-6 | 2.1Quadratic Relationships | Discussion on quadratic relationships with reference to real life scenarios e.g. throwing a ball straight up.Features of the graph $y=x^{2}$* Shape
* Axial intercepts
* Turning points
* Equation of axis of symmetry

Quadratics in each of the following forms are explored * General $y=ax^{2}+bx+c, a\ne 0$
* Factored $y=a\left(x-α\right)\left(x-β\right), a\ne 0$
* Vertex $y=a\left(x-b\right)^{2}+c, a\ne 0$

Students work on the above forms to identify as appropriate axial intercepts, turning points (vertex) and equation of axis of symmetryDetermining the zeros of a quadratic* Factorisation of quadratics from general to factored form
* Use of the quadratic formula (incorporate the meaning of non-real zeros)

Complete the square to determine turning point of a quadratic |  |
| 1-7 | Investigate the discriminant, $∆, $of a function and its significance for the number and nature of the zeros of the graph of the function.* If $∆<0$, two non-real distinct zeros, distinction between rational and irrational zeros
* If $∆ =0$, real repeated zero
* If $∆>0$, two real distinct zeros, distinction between rational and irrational zeros

Relationship between the leading coefficient of a quadratic and its discriminant for positive definite and negative definite quadraticsThe sum and product of real zeros* Revision of surds

Determining quadratic functions from given zeros and a point on the quadraticQuadratic modelling* Determining variables such as height or time from a quadratic
* Optimisation problems such as perimeter dimensions for maximum area
 | **INVESTIGATION**Features of Polynomials |
| 1-8 |
| **EASTER** |
| 1-9 | 2.2Cubic and Quartic Polynomials | Definition of a cubic Terminology, degree and forms* General $y=ax^{3}+bx^{2}+cx+d, a\ne 0$
* Point of Inflection $y=a(x-b)^{3}+c, a\ne 0$
* Factored $y=a\left(x-α\right)\left(x-β\right)\left(x-γ\right), a\ne 0$

Features* Shape reference to leading coefficient

$$a>0, increasing shape$$$$a<0, decreasing shape $$* Behaviour as $x\rightarrow \pm \infty and y\rightarrow \pm \infty $
* Nature and number of zeros of the graph of a cubic

Explore features of cubics written as a product of:* A linear factor and a quadratic (both real and non-real zeros)
* Three linear factors

Determining cubic functions from given zeros and one other piece of dataDefinition of a quarticTerminology, degree and forms as an extension of the work on cubicsCubic and quartic modelling (using technology)Determining unknown variablesOptimisation such as dimensions for maximum volume |  |
| 1-10 |  | **Revision and SAT 2** **Investigation work and submission**  | **SAT 2 Part 1** – No calculatorSketching graphs, factorising to solve, use of quadratic formula to obtain exact answers**SAT 2 Part 2** - Calculator permitted$∆$, determining quadratics, sum and product, modelling |

**Topic 3 – Trigonometry (6 weeks)**

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| Termweek | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 1-11 | 3.1Cosine and Sine Rules | Right-angled trigonometry* Pythagoras’ theorem
* Trigonometric ratios: sin$θ$, cos$θ$ and tan$θ$

Non-right angled triangle* Cosine rule
	+ Finding the length when given two sides and the included angle
	+ Finding an angle given all sides
 |  |
| 2-1 | * Sine rule
	+ Finding the length of a side when given two angles and one side
	+ Finding the angle given two sides and the non-included angle
* Area of non-right angled triangle

Students complete an assortment of problems involving non-right angled triangles using cosine, sine and area rules |  |
| 2-2 | 3.2Circular Measure and Radian Measure | Introduction to the unit circle and its propertiesHow the unit circle is linked to graphs of $cosθ$ and $sinθ$* The link between the unit circle and $cosθ$, $sinθ$ and $tanθ$ in degrees
* The unit circle definition of $cosθ$, $sinθ$ and $tanθ$ and periodicity using degrees

Definition of radian measure* Conversion between radian and degree measure

Calculation of the lengths of arcs and areas of sectors of circle |  |
| 2-3 | 3.3Trigonometric Functions | Connection between unit circle and $\cos(θ)$, $\sin(θ)$ and $\tan(θ)$ in radians Determine the exact value of cosine and sine from multiples of $\frac{π}{6}$ and $\frac{π}{4}$ using unit circle or graphsMaking the connection that the functions $y=\cos(θ)$ and $y=\sin(θ)$ best describe the horizontal and vertical positions around a circleExplore the features of $y=\sin(θ)$ and$ y=\cos(θ)$* Amplitude $y=A\sin(x)$ and $y=A\cos(x)$
* Period $y=\sin(Bx)$ and $y=\cos(Bx)$
* Phase $y=\sin((x+C))$ and $y=\cos((x+C))$

Solve practical problems in a range of different contexts |  |
| 2-4 | Solve trigonometric equations both algebraically and graphically* Only consider cases such as $cosx=\frac{1}{2}$ and $\sin(\left(2x\right))=\frac{1}{2}$

Special relationships observed of sine and cosine functions * $\sin(\left(-x\right))=-sinx$
* $cos \left(-x\right)=cosx$
* $sin \left(x+\frac{π}{2}\right)=cosx$
* $cos \left(x-\frac{π}{2}\right)=sinx$

Tangent function* Consider the relationship between the angle of inclination and the gradient of a line
* The relationship $tan⁡(x)=\frac{sin⁡(x)}{cos⁡(x)}$
* Graphs of the functions
	+ $y=tanx$
	+ $y=tanBx$
	+ $y=tan⁡(x+C)$
 |  |
| 2-5 |  | **Revision and SAT 3** | **SAT 3**1 hourEntire topicCalculator permitted |
| 2-6 |  | **EXAMINATION REVISION** |  |
| 2-72-8 |  | **YEAR 11 EXAMS**  | End Semester One |