**Stage 2 Essential Mathematics**

**Exemplar Open Topic 6: Optimisation**

**This topic can replace Topic 1: Scales, Plans and Models, or Topic 3: Business Applications**

The focus of this topic is on the many aspects of everyday life and business management where efficiency and the finding of optimal strategies are significant considerations. The decisions that individuals and organisations make are often driven by determining the shortest, cheapest, most profitable etc. ways of completing a task.

In this topic students study a range of network problems, which can be represented by graphical means as a network. Algorithms are developed that enable them to find the number of paths, shortest or longest path, and minimum connection or maximum flow in a network.

Students are encouraged to investigate the effects of changing the initial parameters of problems with a view to improving the solutions. For instance, in a critical path analysis, which jobs could be shortened to improve the minimum completion time? In a problem of maximum flow, which connections could be added or upgraded to improve the flow?

The arithmetic computations required for the solution of the problems presented in this topic can be conducted without electronic technology.

Subtopic 6.1: Networks

| Key Questions and Key Concepts | Considerations for Developing Teaching and Learning Strategies |
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| What are networks and what information do they provide?   * Reading information from a network diagram | Students examine network diagrams drawn from a variety of contexts. They interpret information presented in both weighted and directed network diagrams and answer specific questions about contextual situations. Networks to be considered could include the distance between nodes, time of travel, direction of travel, capacity of an arc. |
| * Using appropriate terminology | The correct terminology is taught where it is relevant to the problems studied (e.g. arcs, nodes, directed and undirected networks, trees, circuits) to enable consistent and concise communication in the discussion of networks and network problems. |
| How can networks be used to represent situations in which there is a problem to be solved?   * Connectivity networks * Flow networks | Students are assisted to see how the information in problems can be represented in network form.  For example:   * We have to drive between two given places. Which is the best way to go? Why? * The local council is planning road upgrades because a lot of traffic passes through our area on its way to somewhere else. Which are the best roads to upgrade, and why? * Students have asked for drinking fountains to be installed in specific locations at the school. What is the best way to connect them all to the rainwater supply? |
| How many paths are there through a directed network?   * With and without restrictions | By beginning with using trial and error to find the number of paths through a directed network, students appreciate the efficiency of using the algorithm. The problems are extended to include restrictions. For example, avoiding a node or an arc (e.g. because of an accident or a burst water main) or having to use a specified node or arc (e.g. because someone has to be picked up on the way). |
| What is the shortest or longest path through a network?   * With and without restrictions | Students consider weighted networks where each arc incurs a ‘cost’ or ‘profit’. They explore the idea of an optimal or shortest path which uses the least cost or gains the most profit (with and without restrictions) and apply the algorithm for finding it. They interpret the meaning and understand the limitations of the answers gained using this kind of simplified mathematical model. |
| What is the cheapest way to connect up a set of points if there is more than one option available?   * Minimum spanning tree problems | Explore the idea of a tree being a connected network with no circuits (i.e. no redundancy).  Students explore both scaled and unscaled representations of minimum spanning tree problems when seeking a solution. More than one algorithm is available and students consider which might be best in a given situation.  Extensions to these problems take practical considerations into account. For instance:   * What if a connection cannot be made in a straight line? * What happens to the best solution if extra nodes are connected to the system later? * Is the optimal solution practical if there are limitations on how far any node in the network can be from a source node? |
| What is the maximum flow that can be achieved through a network of conduits?   * Use of an algorithm to find maximum flow | The flow considered could be freight, people, water, telephone calls, Internet connections, or traffic.  The algorithm using the exhaustion of paths is easier than the Dedekind ‘cuts’ method for all but the simplest networks. The cuts method is, however, useful when considering upgrades to a system of flow. Extensions of these problems would deal mainly with upgrading a system by either creating a new connection or improving an existing one. |

**Subtopic 6.2: Critical Path Analysis**

| **Key Questions and Key Concepts** | **Considerations for Developing Teaching and Learning Strategies** |
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| If a job requires the completion of a series of tasks with set precedence, what is the minimum time in which this job can be finished? | Students become acquainted with the idea of precedence in the flow of jobs that make up a complex task. Through a practical example, they explore how to create a precedence table that indicates which other jobs must be completed before a given job can start. |
| * Precedence tables * Drawing networks | From a precedence table, a network can be drawn to represent the task. For straightforward networks this can be done by trial and error; however, students may benefit from being taught how to use a bipartite graph to work out the order in which to construct the nodes. |
| * Dummy links | It is sometimes necessary to use ‘dummy’ arcs in the network to show a given precedence correctly (e.g. when job *E* requires both *A* and *B* to be complete but job *C* requires only *B* to be complete). Students gain an understanding that two different-looking networks may be topologically identical. |
| For which of the tasks is it critical that there is no delay?   * Forward and backward scan * Minimum completion time * Critical path * Earliest and latest starting times for individual tasks * Slack time | Once a network representation is available for a problem, students can determine the minimum completion time and critical jobs by finding the longest path through the network. They discuss the amount of leeway available in the starting time for a given job in the network, and what happens if time for a specific job is shortened or lengthened. They look for ways of reducing the minimum completion time in the context of a specific problem. Students discuss the reasonableness of their results and any limitations to the model in the context of the problem. |